Section 1.3 Continuity (Minimum Homework: 1, 3, 5, 7, 9, 11, 13, 17, 19, 25, 27)

We cover the concept of continuity at specific values of $x = a$ in this section.

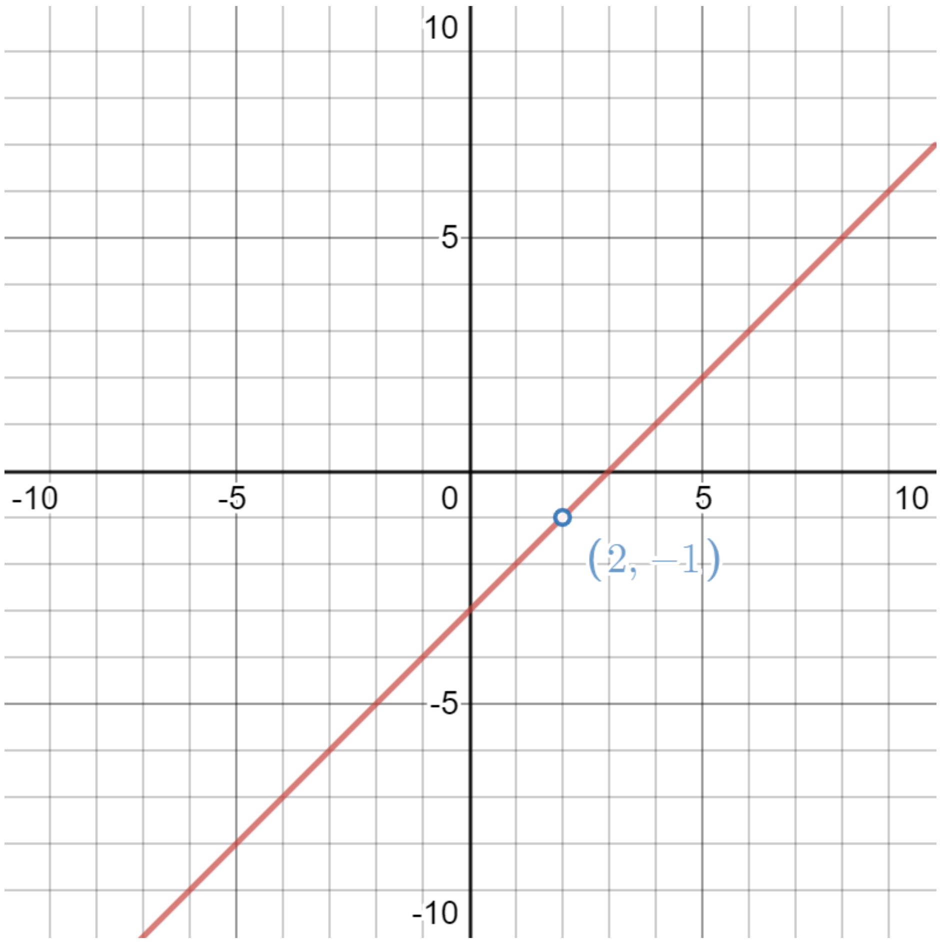
Informally a function is **NOT CONTINOUS** (or we can say the function is discontinuous) at a value of $x = a$ if one of three things happens on the graph of the function for that value of $x = a$.

- 1) There is a hole in the graph of the function, and the function is undefined at that value of x.
- 2) There is a hole in the graph of the function at $x = a$ and the function is defined at the value of x.
- 3) There is a jump in the graph at the value of $x = a$.
- 4) There is a vertical asymptote at the value of $x = a$.

Case 1 example: The function has a **HOLE** at $x = 2$ and the function **IS NOT DEFINED** at $x = 2$ as there is not a solid circle above or below the hole.

We can say the following about the function graphed below:

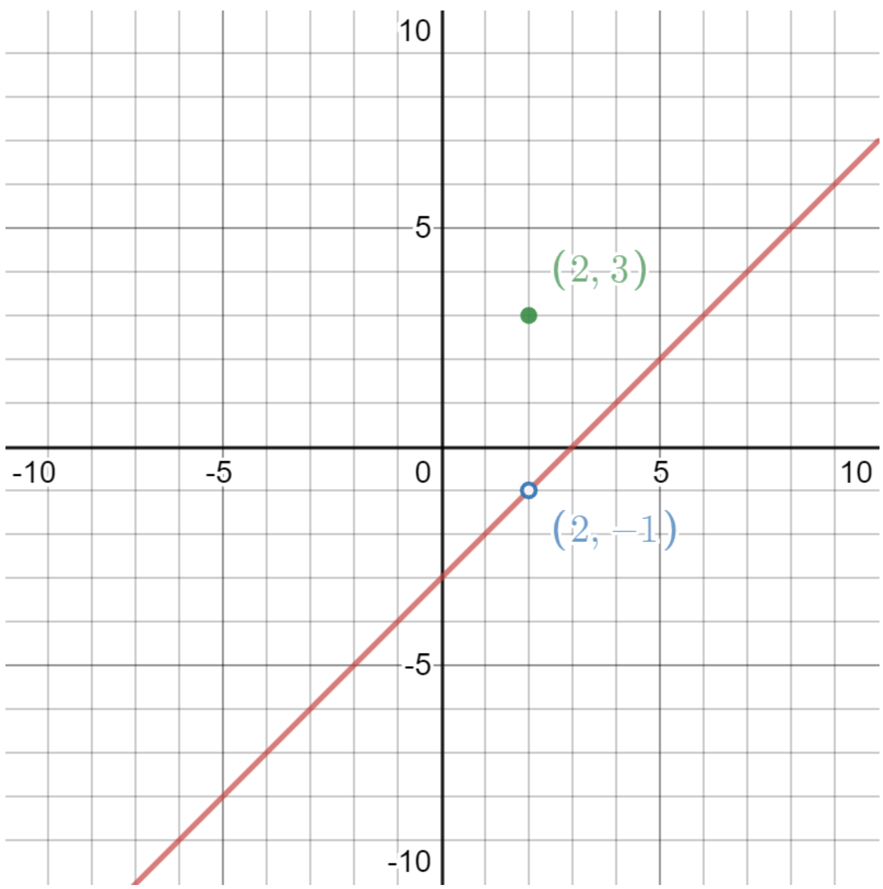
- Is not continuous at $x = 2$ Informal reason: there is a hole at $x = 2$ and the function is undefined at $x = 2$



Case 1 example: The function has a **HOLE** at $x = 2$ and the function **IS DEFINED** at $x = 2$ as there is a solid circle above the hole.

We can say the following about the function graphed below:

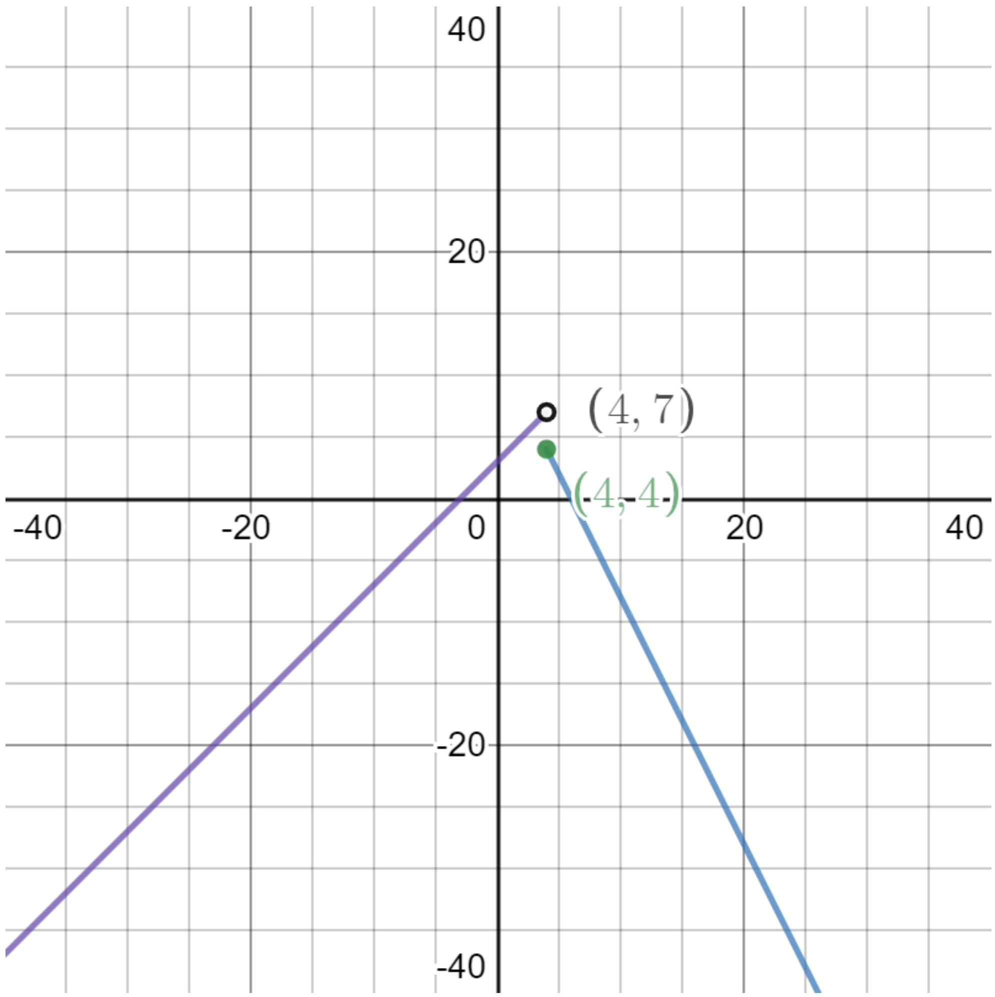
- Is not continuous at $x = 2$: Informal reason, there is a hole at $x = 2$ and the function is defined at $x = 2$.



Case 3 example: The function has a **JUMP** at $x = 4$

We can say the following about the function graphed below:

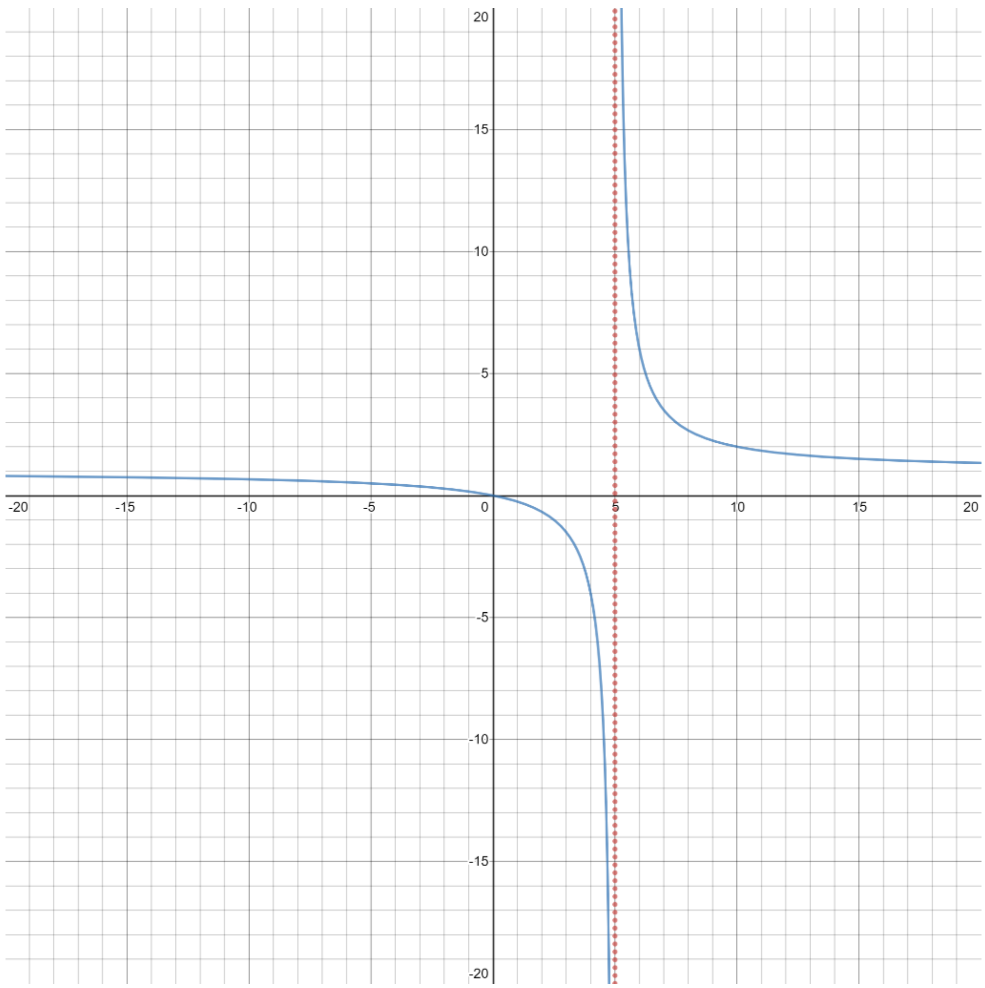
- Is not continuous (discontinuous) at $x = 4$ Informal reason – there is a jump at $x = 4$.



Case 4 example: The function has a **VERTICAL ASYMPTOTE** at $x = 5$

We can say the following about the function graphed below:

- Is discontinuous at $x = 5$, informal reason – there is an asymptote at $x = 5$



Formally a function f is **NOT CONTINOUS** at a point x=a, when one or more of the following occurs.

- 1) The function f is **NOT DEFINED** at $x = a$
  This happens in:
  Case 1 ("unplugged hole")
  Case 4 (vertical asymptote)

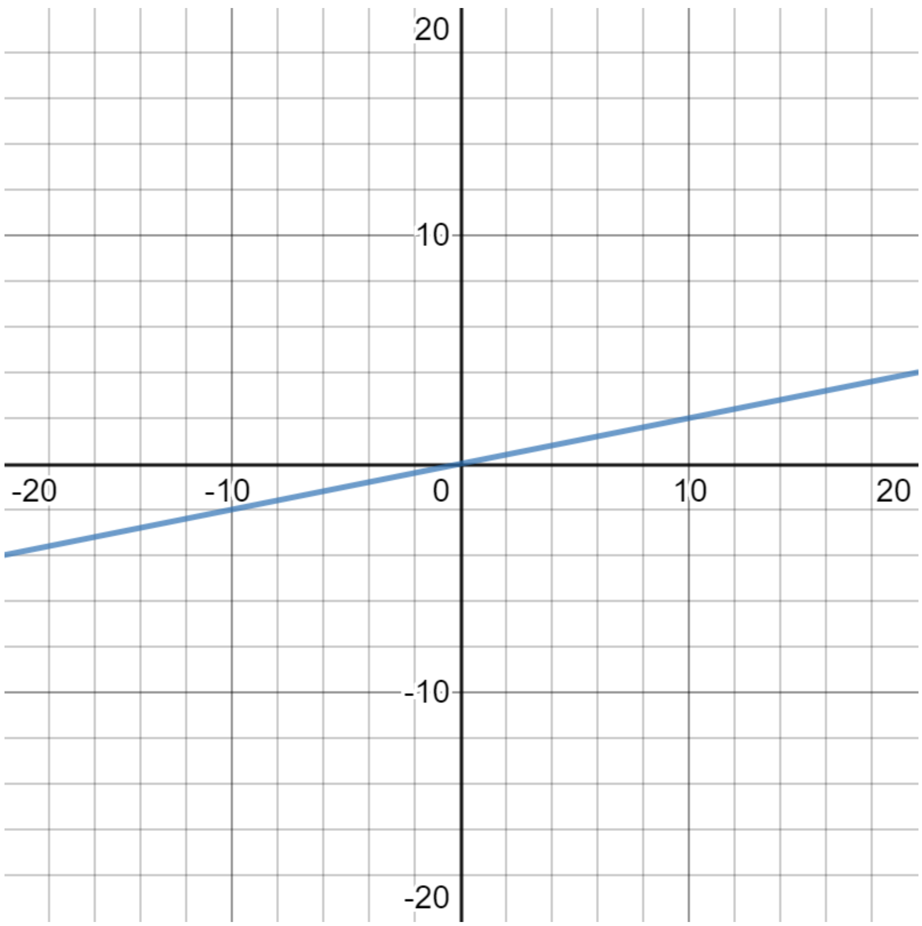
- 2) The limit at $x = a$ exists but the function value is not equal to limit value.
  This happens in:
  Case 2 ("plugged hole")

- 3) The function is defined at $x = a$, but the two-sided limit does not exist at $x = a$.
  This happens in:
  Case 3 (Jump)

We say a function f is continuous everywhere if it does not have any of the three:

- Hole
- Jump
- Vertical asymptote

This is an example of a function that is continuous everywhere:



Example: Determine all values of $x = a$ where the function $f(x)$ is not continuous.

$f(x) = \frac{x^2+6x-7}{x-1}$

The function is not defined when the denominator equals 0.

$Solve\ x - 1 = 0$

The function is **NOT CONTINOUS** at $x = 1$

Informal reason: There is a hole in the graph of the function at $x = 1$ and the function is undefined at $x = 1$

Graphs of fractions either have a hole or a vertical asymptote for values that make the denominator 0.

- Hole – for the value of x that makes the denominator 0 when the factor CANCELS with the numerator
- Vertical asymptote - for the value of x that makes the denominator 0 when the factor DOES NOT CANCEL with the numerator

$f(x) = \frac{x^2+6x-7}{x-1} = \frac{(x+7)(x-1)}{x-1} = x + 7\ \ (x - 1\ cancels, so\ there\ is\ a\ hole\ at\ x = 1)$

Example: Determine all values of $x = a$ where the function $f(x)$ is not continuous.

$f(x) = \frac{6}{x^2+5x-14}$

The function is not defined when the denominator equals 0.

$Solve\ x^2 + 5x - 14 = 0$
$(x + 7)(x - 2) = 0$
$x = \ -7, 2$

The function is **NOT CONTINOUS** at $x = -7, 2$

Informal reason: There are asymptotes of the graph of the function at $x = -7, 2$

There are vertical asymptotes for both values of x since the factors $(x + 7)\ and\ (x - 2)$ do not cancel with the numerator.

Example: Determine all values of $x = a$ where the function $f(x)$ is not continuous.

$$f(x) = \begin{cases} x - 5, \ if \ x \ \leq 5 \\ 2x - 4, \ if \ x > 5 \end{cases}$$

We need to figure out if there is or is not a jump at $x = 5$.

Plug $x = 5$ into both functions. If the values do not equal, there is a jump at the value of x.

Top function: $f(5) = 5 - 5 = 0$

Bottom function: $f(5) = 2(5) - 4 = 6$

The function has a hole at $x = 5$

The function is **NOT CONTINOUS** at $x = 5$,
informal reason – there is a jump in the graph at $x = 5$.

Example: Determine all values of $x = a$ where the function $f(x)$ is not continuous.

$f(x) = 3x - 12$

The function is **CONTINOUS** everywhere. Unfortunately, there is no Algebra to show this.

Basically these features in an equation can make a function discontinuous at a value of $x = a$.

Holes – denominator equals to zero, and denominator cancels with numerator

Asymptote – denominator equals to zero, and denominator does not cancel with numerator

Jump – function needs to be defined piecewise and the values don't match up at the value of x.

Example: Determine all values of $x = a$ where the function $f(x)$ is not continuous.

$$f(x) = \begin{cases} x - 3, \ if \ x \ \leq 6 \\ 2x - 9, \ if \ x > 6 \end{cases}$$

We need to figure out if there is or is not a jump at $x = 6$.

Plug $x = 6$ into both functions. If the values do not equal, there is a jump at the value of x.

Top function: $f(6) = 6 - 3 = 3$

Bottom function: $f(6) = 2(6) - 9 = 3$

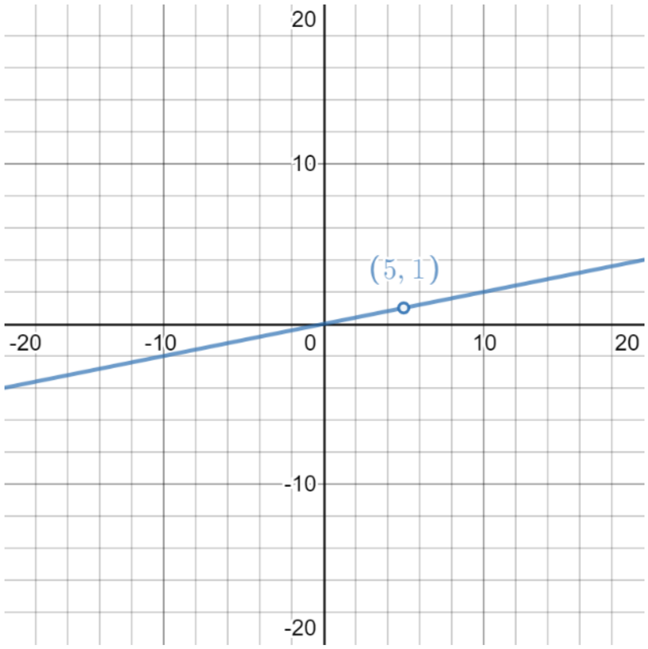
The function does not have a hole at $x = 6$ as the values are equal.

Each piece of the function is a polynomial and polynomials are continuous everywhere.

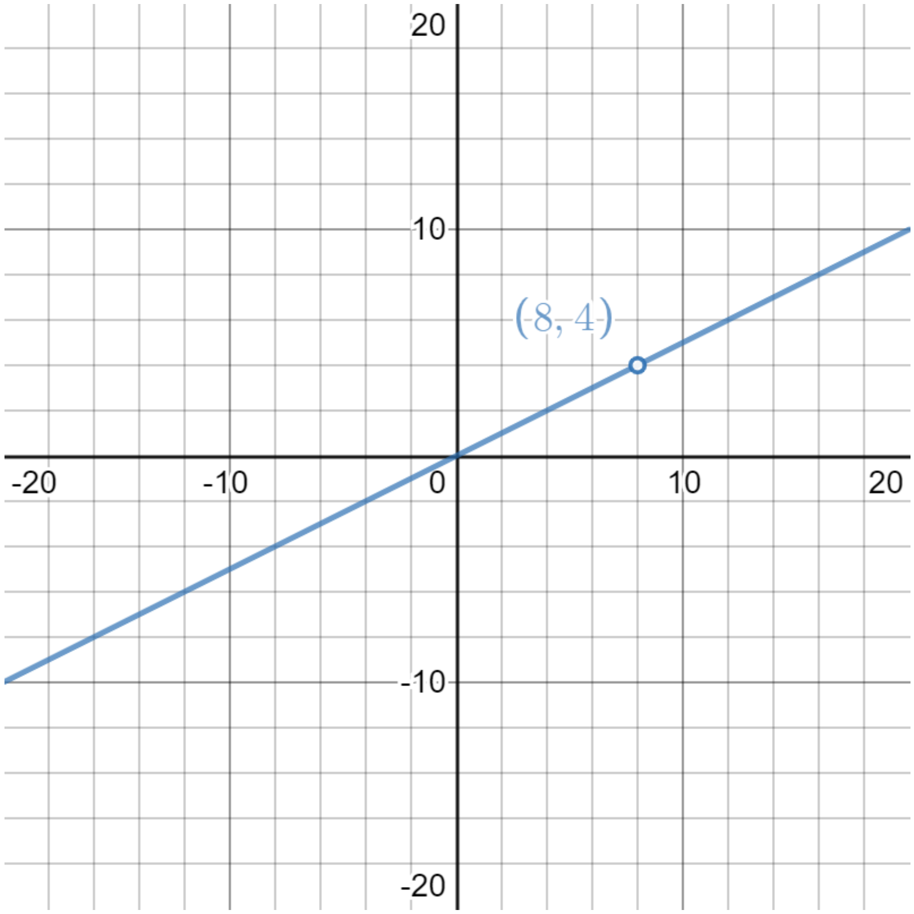
The function is **CONTINOUS** everywhere.

#1-10: Find all values of x = a where the function is discontinuous. State the informal rule that makes the function discontinuous for the value of $x = a$.

1)

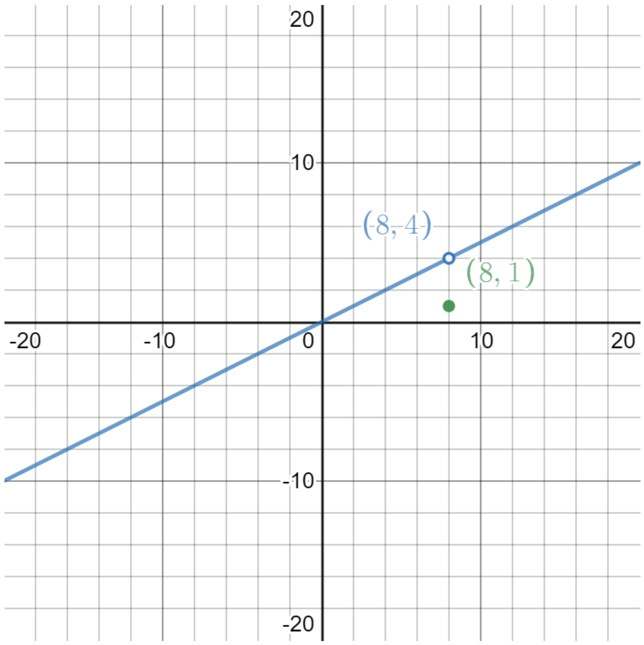


2)

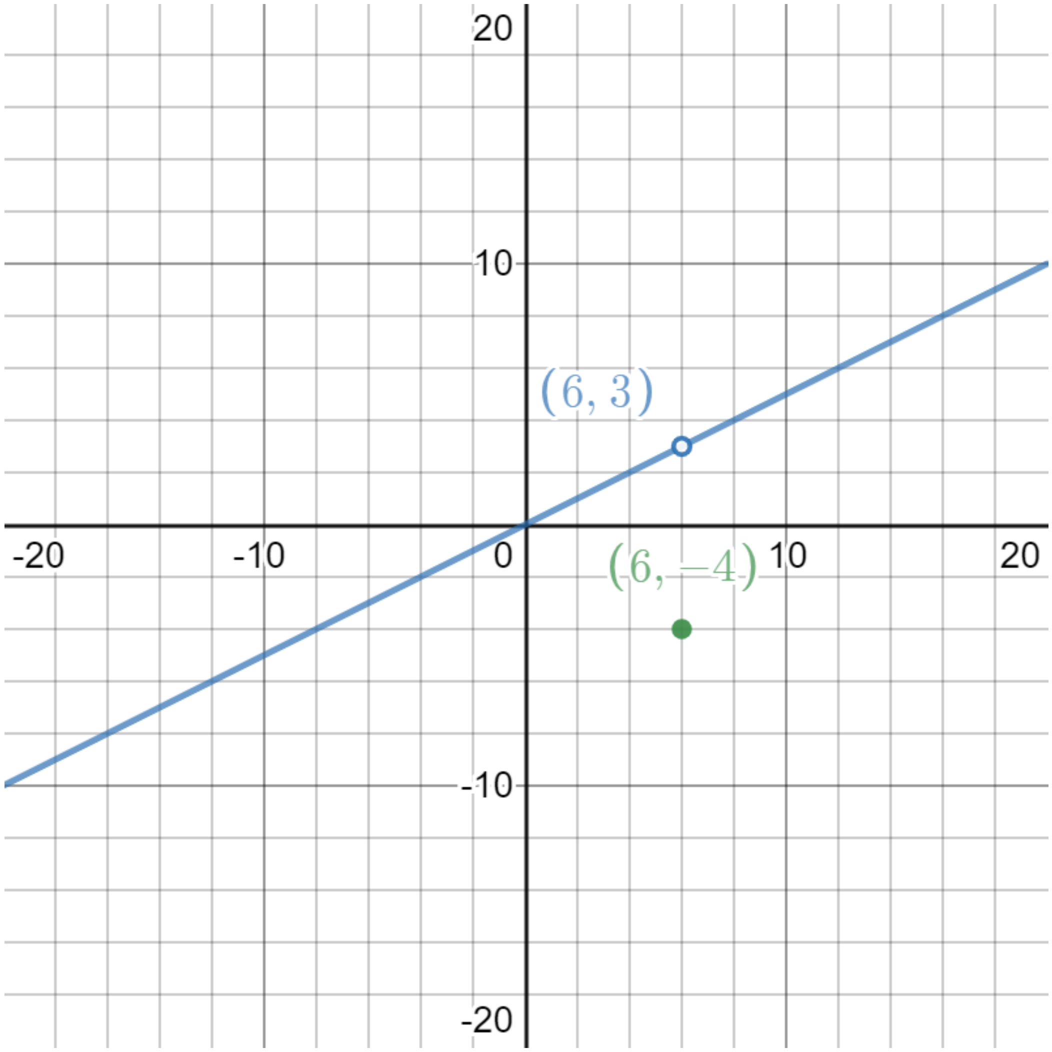


Answer: Not continuous at $x = 8$, (Informal reason: There is a hole in the graph of the function at $x = 8$ and the function is undefined at $x = 8$)

3)

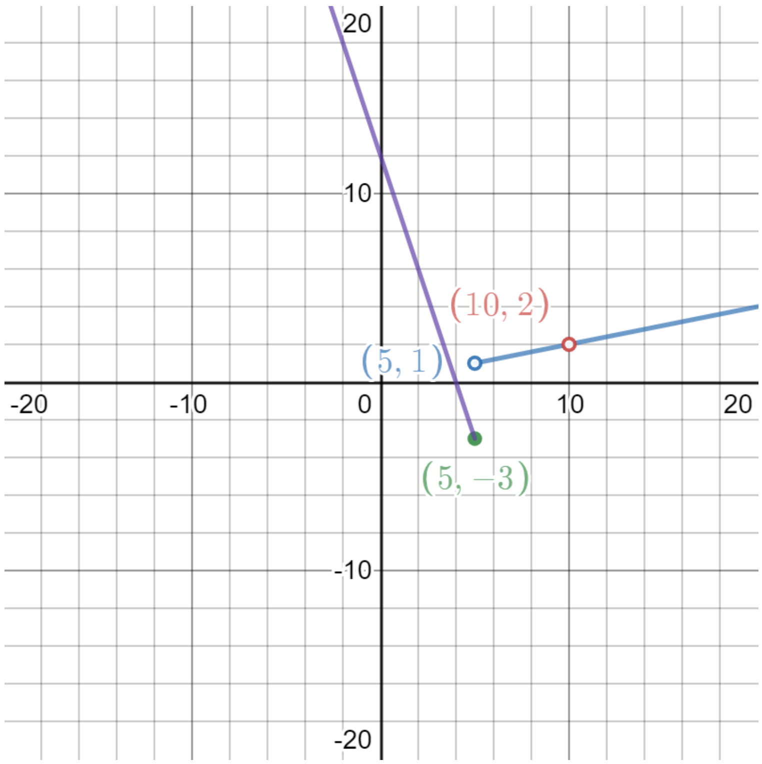


4)

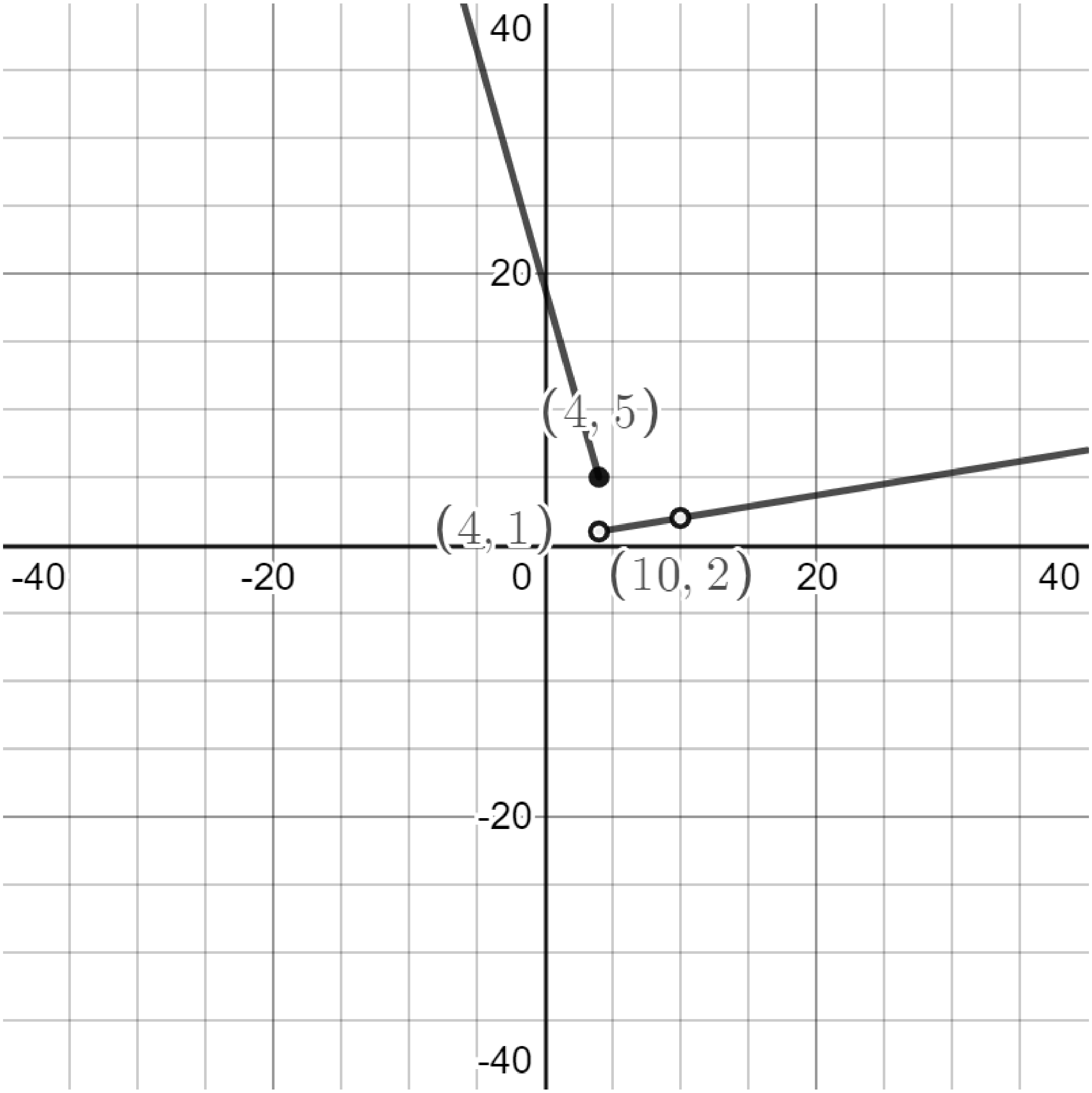


Answer: Not continuous at $x = 6$, (Informal reason: There is a hole in the graph of the function at $x = 6$ and the function is defined at $x = 6$)

5)



6)

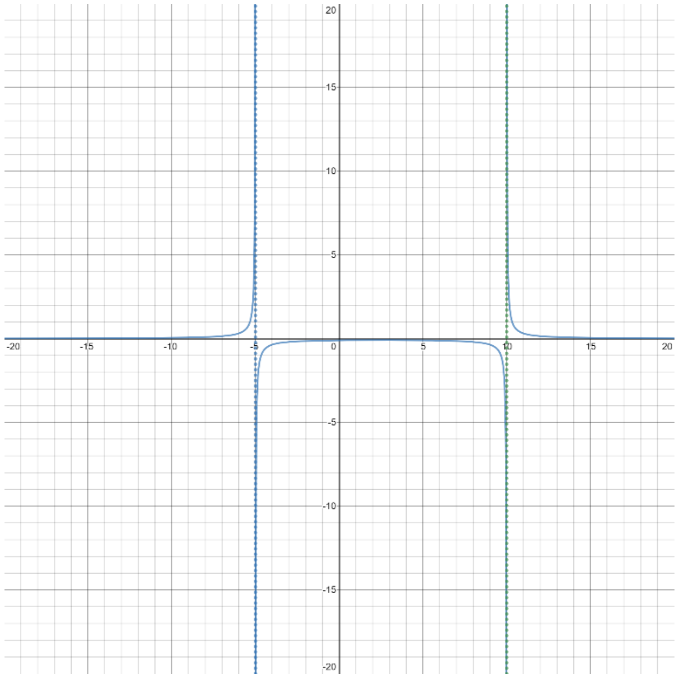


The function is discontinuous at $x = 4\ and\ x = 10$

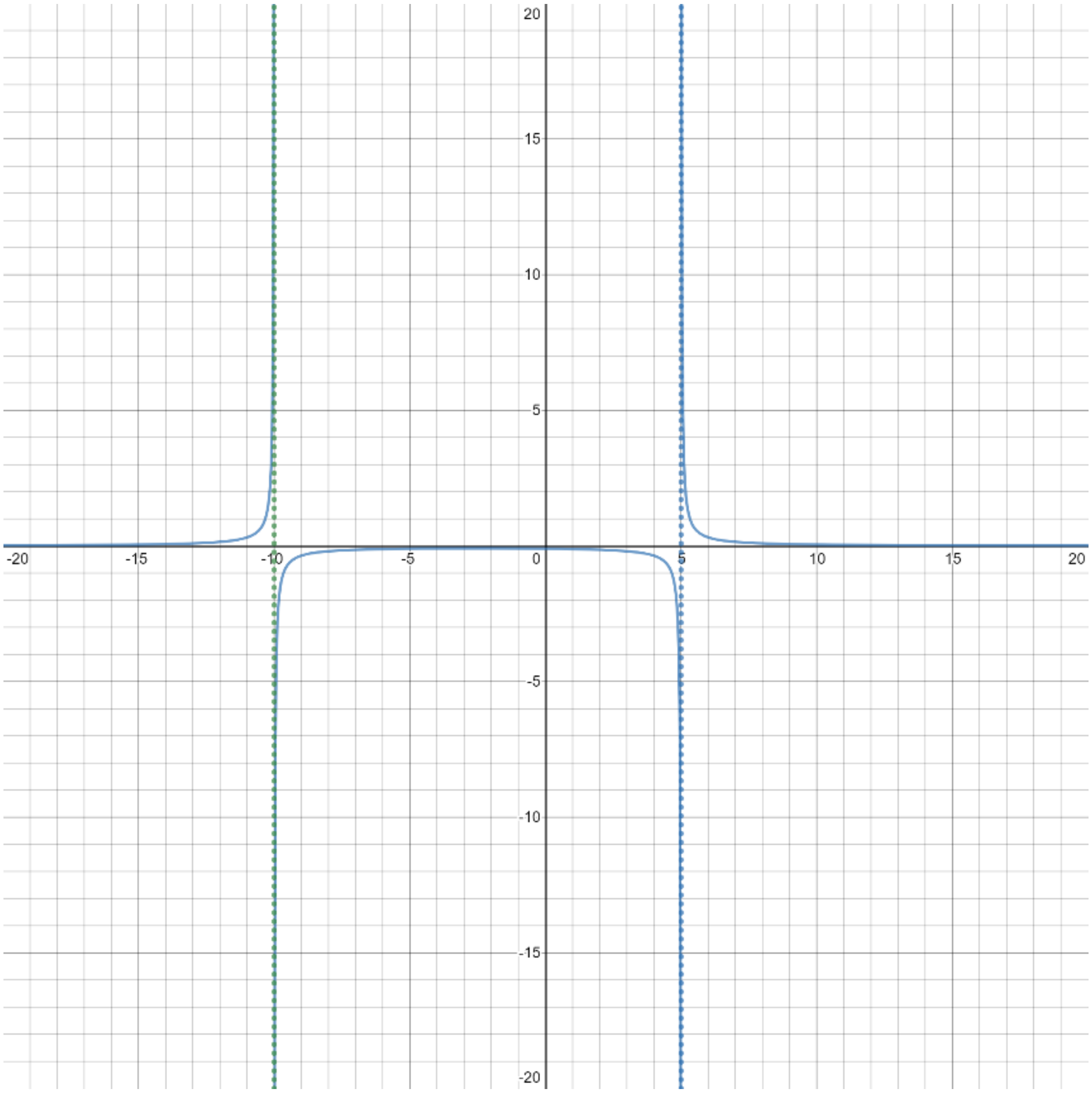
Informal reason: There is a “jump” at $x = 4$.

There is a hole at $x = 10$ and the function is undefined at $x = 10$

7)

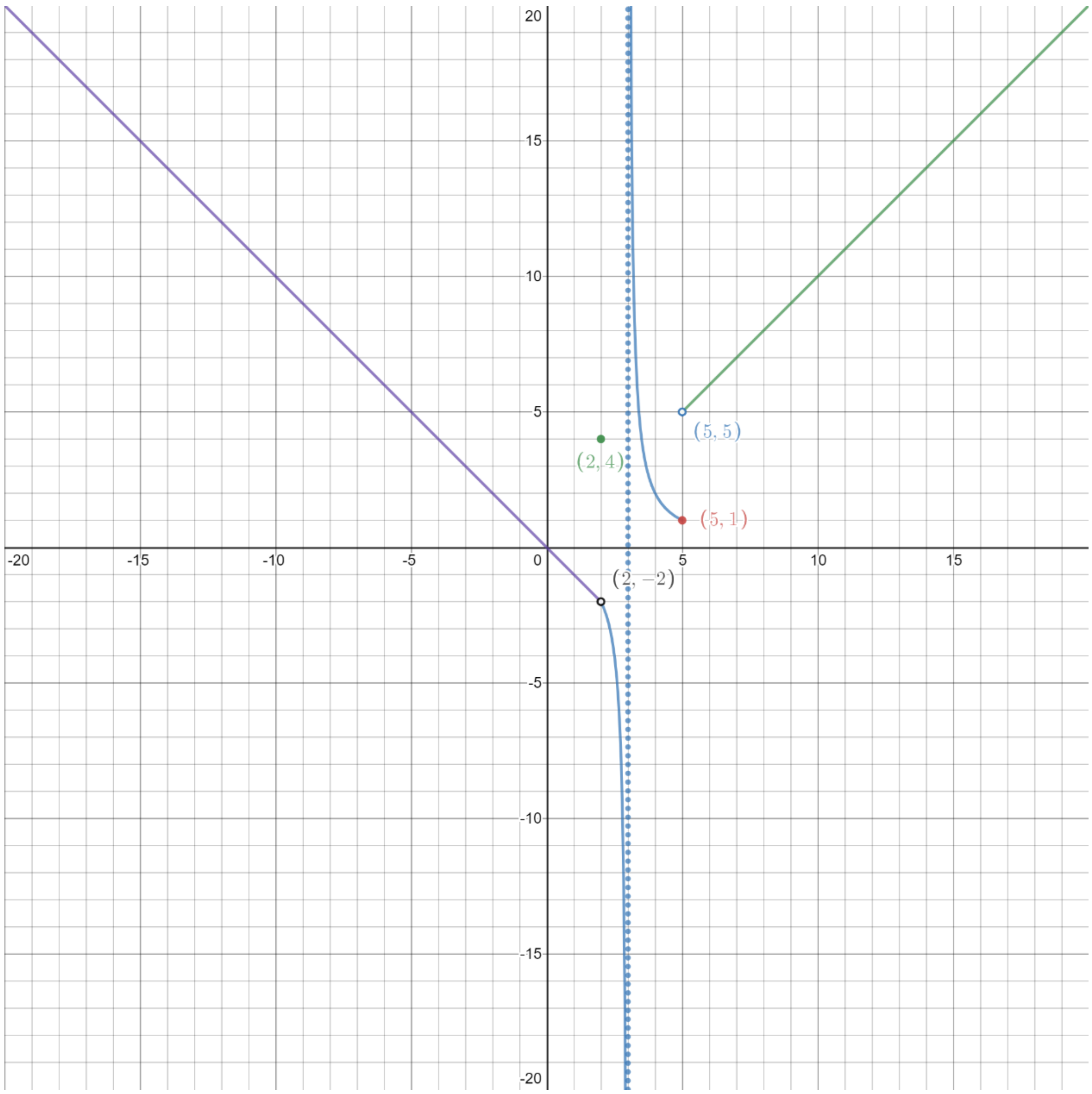


8)

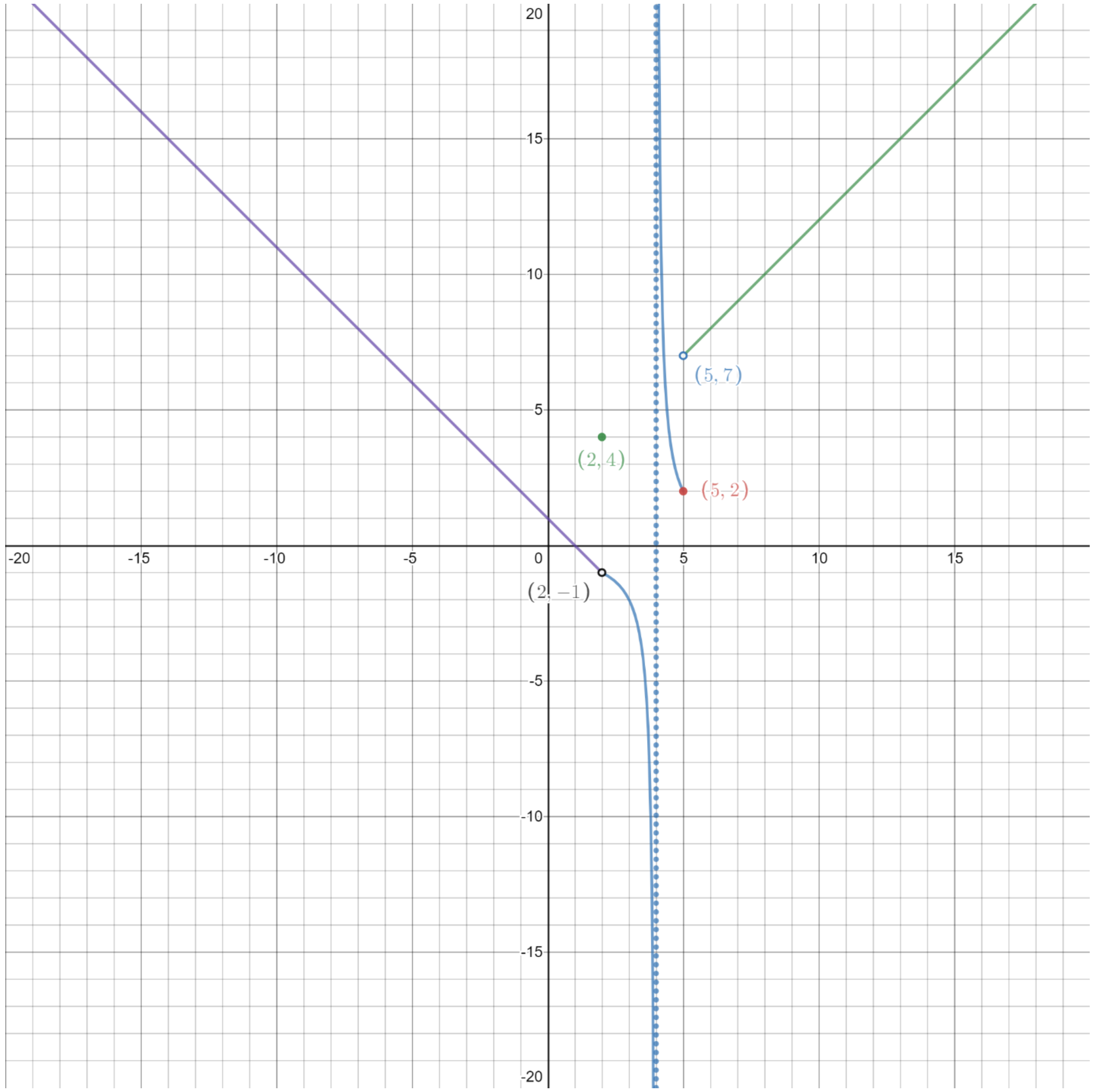


The function in discontinuous at $x = -10 \; and \; x = 5$
Informal reason – There are vertical asymptotes at both $x = -10 \; and \; x = 5$

9)



10)



The function is discontinuous at $x = 2, 4, 5$

Informal reason:

There is a hole in the graph at $x = 2$
and the function is defined at $x = 2$.

There is a vertical asymptote at $x = 4$.
There is a jump at $x = 5$.

#11-30: Find all values of x=a where the function is discontinuous. State the informal reason for each point of discontinuity. State if the function is continuous everywhere.

11) $f(x) = \frac{x-3}{x+4}$

12) $f(x) = \frac{x+1}{x-5}$

Answer: The function is discontinuous at $x = 5$
Informal reason: There is a vertical asymptote at $x = 5$.

13) $f(x) = \frac{x^2+7x+12}{x+3}$

14) $f(x) = f(x) = \frac{x^2+3x+2}{x+1}$

Answer: The function is discontinuous at $x = -1$
Informal reason: There is a hole at $x = -1$

$and\ the\ function\ is\ not\ defined\ at\ x = \ -1.$

15) $f(x) = \frac{x^2-4}{x+2}$

16) $f(x) = \frac{x^2-9}{x+3}$

Answer: The function is discontinuous at $x = -3$
Informal reason: There is a hole at $x = -3$

$and\ the\ function\ is\ not\ defined\ at\ x = -3.$

17) $f(x) = \frac{5}{x^2+3x+2}$

18) $f(x) = \frac{7}{x^2-2x-3}$

The function in discontinuous at $x = -1\ and\ x = 2$
Informal reason – There are vertical asymptotes at both $x = -1\ and\ \ x = 2$

19) $f(x) = 2x - 6$

20) $f(x) = 3x - 2$

Answer: The function is continuous everywhere.

21) $f(x) = x^2 + 6x - 7$

22) $f(x) = x^2 - 4x - 5$

Answer: The function is continuous everywhere.

23) $f(x) = \begin{cases} x+3, & if\ x \le 6 \\ 2x, & if\ x > 6 \end{cases}$

24) $f(x) = \begin{cases} 6x, & if\ x \le 2 \\ 2x+1, & if\ x > 2 \end{cases}$

Answer: The function is not continuous at $x = 2$.

Informal reason: There is a jump in the graph at $x = 2$.

25) $f(x) = \begin{cases} x - 3, & if\ x \le 5 \\ 2x - 9, & if\ x > 5 \end{cases}$

26) $f(x) = \begin{cases} x - 3, & if\ x \le 4 \\ 2x - 9, & if\ x > 4 \end{cases}$

Answer: The function is not continuous at $x = 4$.

Informal reason: There is a jump in the graph at $x = 4$.

27) $f(x) = \begin{cases} x+6, & if\ x \le 6 \\ 2x, & if\ x > 6 \end{cases}$

28) $f(x) = \begin{cases} 6x, & if\ x \le 2 \\ 2x+8, & if\ x > 2 \end{cases}$

Answer: The function is continuous everywhere.

29) $f(x) = \begin{cases} x - 3, & if\ x \leq 5 \\ 2x - 8, & if\ x > 5 \end{cases}$

30) $f(x) = \begin{cases} x - 3, & if\ x \leq 4 \\ 2x - 7, & if\ x > 4 \end{cases}$

Answer: The function is continuous everywhere.